P. 392-394
3-6
9.3

3) a.) $t=0$ population of 5000 growing at a rate of 300 people/year

$$\boxed{P = 5000 + 3t}$$

b.) $t=0$ population of 3200 growing at annual rate of 4% per year

$$\boxed{P = 3200(1.04)^t}$$

c.) Population oscillates over 5-year period between low of 1200 in year $t=0$ and a high of 3000

$P = \frac{2\pi}{B}$ $\quad 5B = 2\pi$ $\quad Amp = \frac{3000-1200}{2}$

$5 = \frac{2\pi}{B}$ $\quad B = \frac{2\pi}{5}$ $\quad A = 900$

Midline: $y = \frac{3000+1200}{2}$

$y = 2100$

Reflected cosine (Starts down)

$$\boxed{P(t) = -900\cos\left(\frac{2\pi}{5}t\right) + 2100}$$

4.) $R = f(t) = -5000\cos\left(\frac{\pi t}{6}\right) + 10{,}000$

a.) Evaluate and interpret $f(3) - f(2)$ where t = months since Jan 1

$-5000\cos\left(\frac{3\pi}{6}\right) + 10{,}000 - \left(-5000\cos\left(\frac{2\pi}{6}\right) + 10000\right)$

$-5000(0) + 10{,}000 - \left(-5000\left(\frac{1}{2}\right) + 10{,}000\right)$

$10{,}000 - 7{,}500 = \boxed{-2{,}500}$

It means the number of rabbits will in rease by 2500 from March 1 to April 1.

4b.) SOLVE AND INTERPRET:

$f(t) = 12{,}000 \quad 0 \le t \le 12$

$12{,}000 = -5000\cos\left(\frac{\pi t}{6}\right) + 10{,}000$

$2{,}000 = -5000\cos\left(\frac{\pi t}{6}\right)$

$-\frac{2}{5} = \cos\left(\frac{\pi t}{6}\right)$

$\frac{\pi t}{6} = \cos^{-1}\left(-\frac{2}{5}\right)$

$\frac{\pi}{6}t = 1.982$ and $\frac{\pi}{6}t = 2\pi - 1.982$

$$\boxed{t = 3.786 \text{ and } t = 8.214}$$

The population reaches 12,000 in late April ($t = 3.786$) and again in early September.

5.) $f(t) = mt + b + A\sin\left(\frac{\pi t}{6}\right)$ $\quad t$ month Since Jan 1

t	0	3	6	9	12 = t
Date	Jan 1	Apr 1	July 1	Oct 1	Jan 1
Price	\$20	\$37.50	\$35	\$32.50	\$50

a.) $(0,20)$ $(12,50)$ $m = \frac{50-20}{12-0} = \frac{30}{12} = \frac{5}{2} = 2.5$

b (y-int) $\quad$ (Slope)

$$\boxed{m = 2.5 \quad b = 20}$$

Solve for A using any point: $(3, 37.50)$

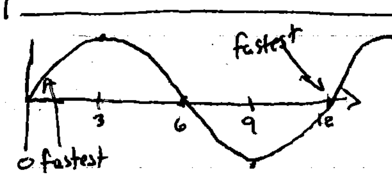
$37.50 = 2.5(3) + 20 + A\sin\left(\frac{\pi 3}{6}\right)$

$37.50 = 27.50 + A\sin\left(\frac{\pi}{2}\right)$

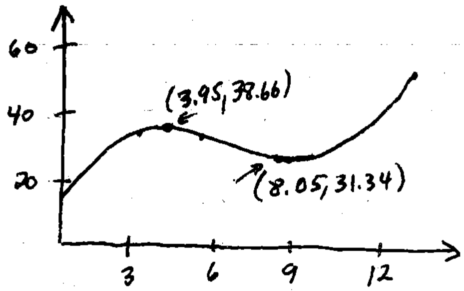
$10 = A \cdot 1 \quad \boxed{A = 10}$

$f(t) = 2.5t + 20 + 10\sin\left(\frac{\pi t}{6}\right)$

b.) The stock will appreciate (gain value) the most when the sine function increases the fastest, at $t=0$ and $t=11$ (January + December).



c.) If the function is graphed on calculator, you can identify where it is increasing and decreasing.



It is decreasing from $t = 4$ (3.95) to $t = 8$ (8.05) or from May to September.

6.) Power Company A: min at midnight 40 megawatts, max at noon of 90 mw, back to 40 at midn

a.) Amplitude $= \frac{90-40}{2} = 25$ Midline: $y = \frac{90+40}{2} \Rightarrow y = 65$ Period $= 24 = \frac{2\pi}{B}$ $24B = 2\pi$

$B = \frac{\pi}{12}$

$\boxed{f(t) = -25\cos\left(\frac{\pi}{12}t\right) + 65}$

b.) Power Company B: $g(t) = 80 - 30\sin\left(\frac{\pi}{12}t\right)$

$\boxed{\text{Amplitude} = 30}$ Period $= \frac{2\pi}{\frac{\pi}{12}} = 2\pi \cdot \frac{12}{\pi} = \boxed{24}$

The amplitude of 30 means it increases 30 from the midline to a max of 110 and decreases 30 to a minimum of 50.
The period of 24 means the cycle will repeat every day.

c.) Graph and find t such that $f(t) = g(t)$ $0 \le t < 24$

The graphs intersect at $\boxed{t = 4.160 \text{ and } t = 13.148.}$

The amount of power required by each city is the same around 4AM and 1PM.

d.) The power company should be interested in the maximum value of $h(t) = f(t) + g(t)$ because it will be the maximum amount of power they need to supply to both cities.

The max value (184.051 MW) occurs at $t = 15.346$ or 3:21.

e.) $h(t)$ as a single sine function:

$h(t) = f(t) + g(t)$

$= 65 - 25\cos\left(\frac{\pi}{12}t\right) + 80 - 30\sin\left(\frac{\pi}{12}t\right)$

$= 145 - 30\sin\left(\frac{\pi}{12}t\right) - 25\cos\left(\frac{\pi}{12}t\right)$

$a_1 = -30 \quad a_2 = -25$

$A = \sqrt{a_1^2 + a_2^2}$

$= \sqrt{900 + 625}$

$A = \sqrt{1525} \approx 39.051$

$\phi = \tan^{-1}\left(\frac{a_2}{a_1}\right)$

$= \tan^{-1}\left(\frac{-25}{-30}\right)$

$\phi = \tan^{-1}\left(\frac{5}{6}\right) = .6947 + \pi$

$\cos\phi = \frac{a_1}{A} = \frac{-30}{\sqrt{1525}}$ $\ominus$

$\sin\phi = \frac{a_2}{A} = \frac{-25}{\sqrt{1525}}$ $\ominus$

$^*\underline{Q3}$!!

$\boxed{h(t) = 145 + 39.051\sin\left(\frac{\pi}{12}t + 3.8363\right)}$

-or-

$\boxed{h(t) = 145 + \sqrt{1525}\sin\left(\frac{\pi}{12}t + 3.8363\right)}$

or

(Factor out -1)

$h(t) = 145 - \left[30\sin\left(\frac{\pi}{12}t\right) + 25\cos\left(\frac{\pi}{12}t\right)\right]$

$a_1 = 30 \quad a_2 = 25$

$A = \sqrt{900 + 625}$

$A = \sqrt{1525}$

$\phi = \tan^{-1}\left(\frac{25}{30}\right)$

$\phi = \tan^{-1}\left(\frac{5}{6}\right) = .6947$

$\cos\phi = \frac{30}{\sqrt{1525}}$ $\oplus$

$\sin\phi = \frac{25}{\sqrt{1525}}$ $\oplus$

$\underline{Q1}$

-or-

$\boxed{h(t) = 145 - \sqrt{1525}\sin\left(\frac{\pi}{12}t + .6947\right)}$

Exact MAX: $145 + \sqrt{1525}(1) = \boxed{145 + \sqrt{1525}}$

Max value of sine

or

$145 - \sqrt{1525}(-1) = 145 + \sqrt{152}$

min value of sine